Scholarship Skills 2020

Revise Mathematics

Exercise due Monday, 3rd February 2020

Apply what you've learned about writing mathematics to rewrite this proof. Don't be afraid to *rewrite* it, rather than tinker about with it in small ways. The LaTeX source for this proof is on the web site, so you can edit it to create your own version.

The Largest Prime

Suppose that there were a largest prime number p_i. Then consider the product $\prod_{j=0}^{p_i-1} p_i - j$. Then $\left(\prod_{j=0}^{p_i-1} p_i - j\right) + 1$ cannot be divided evenly by any of the numbers up to p_i, $2, 3, 4, \ldots, p_i$ because each of these divides the left factor evenly, but not the right factor, hence not their sum. (Recall that if a_1 divides a_2 and $a_2 = a_3 + a_4$ then if a_1 divides a_3, it will also divide a_4.) Since we are assuming p_i is the largest prime, $\left(\prod_{j=0}^{p_i-1} p_i - j\right) + 1$ can have no prime factors greater than p_i, hence $\left(\prod_{j=0}^{p_i-1} p_i - j\right) + 1$ is a prime, and it is greater than p_i, since $\prod_{j=0}^{p_i-1} p_i - j \geq p_i$. This contradicts the maximality of p_i. Hence the assumption that p_i is the largest prime must be false, and so there is no largest prime.