CFL Big Picture

Context Free Languages Conclusion

- We have studied the class of context free languages (CFL)
- We saw two different ways to express a CFL
  1. Context Free Grammar
  2. Push Down Automata

- We showed that some were equally expressive
  – We need non-deterministic PDA to express Context Free Grammars
  – Recall the construction of the PDA had only one state, and possible several transitions on the same Non-terminal.
- Some were easier to use than others to describe some languages

Acceptance

- Context free grammars

  The *language of the CFG* , G, is the set

  $L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$ where

  S is the start symbol of G

  $\Rightarrow$ is the single step relation between derivations

- Push down automata
  - Use of instantaneous descriptions (IDs) and the relation |- between IDs
  - Acceptance by final state
  - Acceptance by empty stack

Algorithms

- We studied algorithms to transform one description into another
  1. Context Free Grammar to PDA (Theorem 2.21 pg 115)
  2. PDA into Context Free Grammar (Lemma 2.27 pg 119)
- We studied how to transform grammars
  1. To remove ambiguity (layering)
     1. Non-ambiguous languages can have ambiguous grammars
  2. To transform into Chomsky Normal Form

Properties

- We saw that Regular Languages have many properties

- Closure properties
  - Union
  - Kleene – star
  - Intersection
  - Complement
  - Reversal
  - Difference

CFL Languages have fewer properties

- Closure properties
  - Union
  - Kleene – star
  - Concat

- But we do have the intersection between CFL and RL produces a CFL

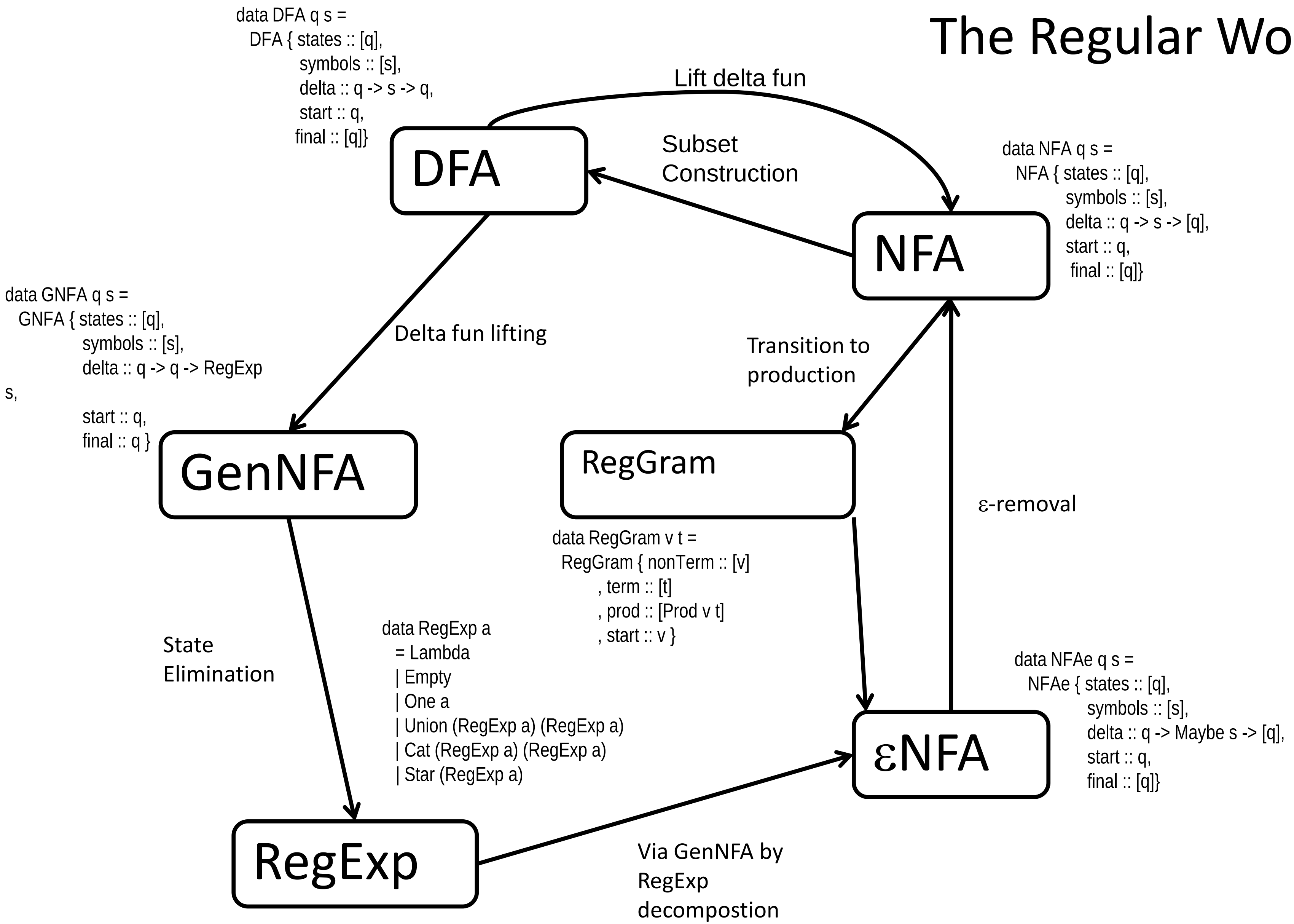
Proving some language is not CF

- Pumping lemma for CF languages
- Let L be a CFL. Then there exists a number n (depending on L) such that every string w in L of length greater than n contains a CFL pump.

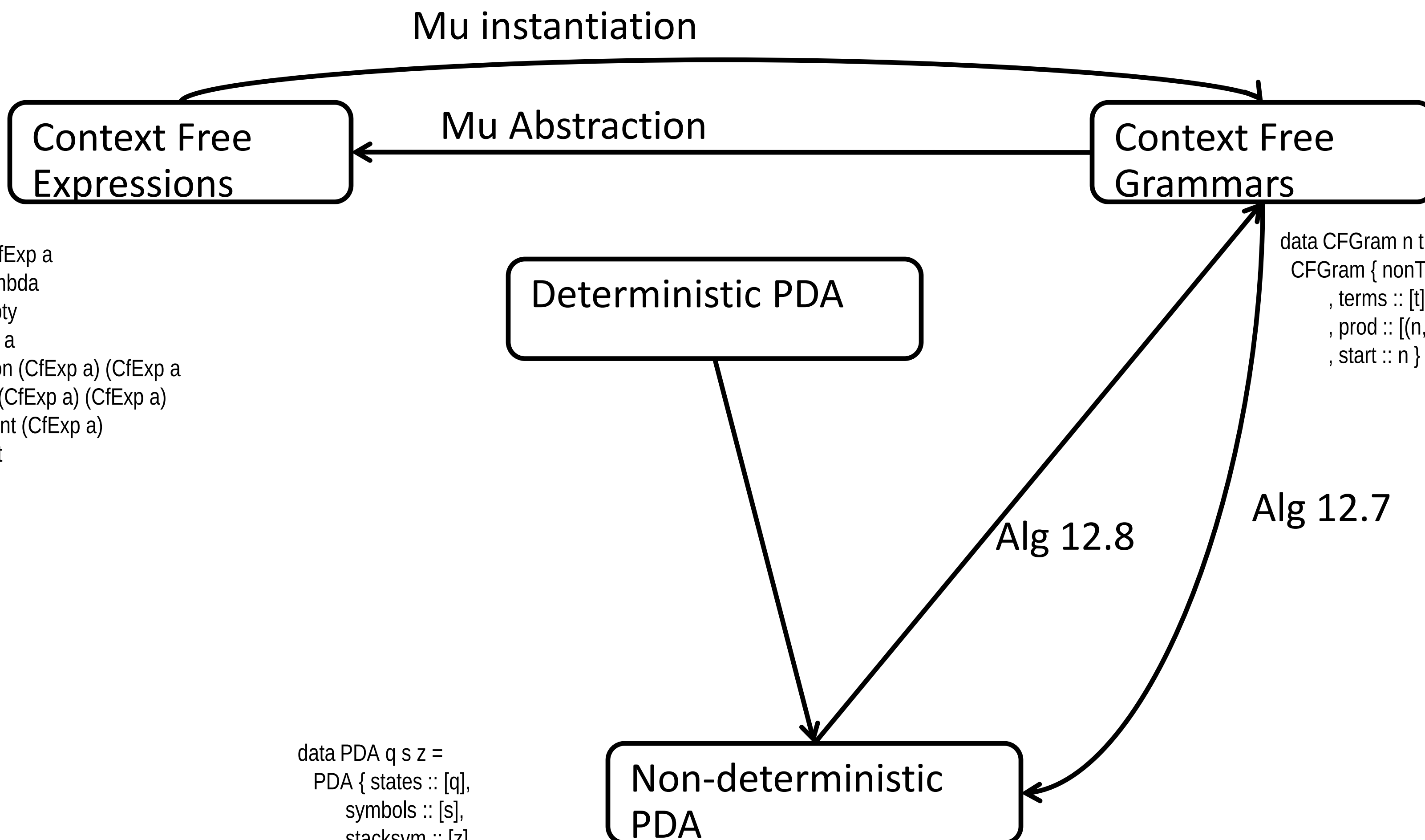
Context Free Pump

- A *CFL pump* consists of two non-overlapping substrings that can be pumped simultaneously while staying in the language.
- Precisely, two substrings u and v constitute a CFL pump for a string w of L ($|w| > m$) when
  1. $uv \neq \Lambda$ (which means that at least one of u or v is not empty)
  2. And we can write $w=xuyvz$, so that for every $i \geq 0$
  3. $xu^iyv^iz \in L$

The Regular World



The Context Free World




```
data CfExp a
  = Lambda
  | Empty
  | One a
  | Union (CfExp a) (CfExp a
  | Cat (CfExp a) (CfExp a)
  | Mu Int (CfExp a)
  | V Int
```

```
data CFGram n t =
  CFGram { nonTerm :: [n]
         , terms :: [t]
         , prod :: [(n,[Sym n t])]
         , start :: n }
```

```
data PDA q s z =
  PDA { states :: [q],
        symbols :: [s],
        stacksym :: [z],
        delta ::  [(q,Maybe s,z,[(q,[z])])],
        start :: q,
        final :: [q]}
```

The Larger World